Randomized algorithms for analysis and control of uncertain systems, R. Tempo, G. Calafiore, F. Dabbene; Springer, Berlin, 2004, ISBN: 1852335246.

Undoubtedly, model uncertainty and robustness have been key themes in the development of modern automatic control during the last four decades. In fact, in many situations feedback control of dynamical systems allows to substantially improve typical control engineering objectives, such as accurate path-following or effective disturbance attenuation, even if only a rather poor mathematical model of the to-be-controlled dynamics is available. On the other hand, optimization-based controller design strategies typically rely on a sufficiently accurate model of the to-be-controlled plant. Recent years have witnessed the development of techniques for quantifying the plant-model mismatch, such as in uncertainty estimation based on measured data or as resulting from model reduction to reduce complexity.

Various widely used paradigms for the mathematical description of parametric and dynamic linear time-invariant uncertainties are rather thoroughly reviewed in the book in Chapter 3. The authors adopt the generalized plant framework and succinctly recapitulate the path from signal and system norms to robust stability and performance characterizations against structured norm-bounded uncertainties on the basis of the structured singular value (Zhou, Doyle, & Glover, 1996), including its relations to the parametric approach (Barmish, 1994). Particular emphasis is laid on the computation of H_2- and H_∞-norms as well as upper bounds of the structured singular value by using linear matrix inequalities (LMIs).

This part on robust stability and nominal performance analysis is complemented by Chapter 4, addressing the LMI-approach to H_∞- and H_2-synthesis, as well as the somewhat more specialized Riccati equation solutions. Both their use within the so-called D/K-iteration for robust controller synthesis, and their relation to LQR and guaranteed-cost control are touched upon. Although these two chapters seem, in particular for the novice readers, somewhat enumerative and technical, the authors' selection of the material and the wealth of references make up an excellent guide through the broad literature on these subjects.

The deficiencies of the classical approaches to robustness analysis are the subject of Chapter 5. Emphasis is put on the fact that an exact computation of robust stability radii against structured uncertainties turns out to be NP-hard in the sense of computational complexity theory. This result prevents the existence of algorithms for testing robust stability or robust performance against structured uncertainties that are in worst-case of polynomial complexity (unless $P = NP$).

In the reviewer's view, this chapter would have benefited from a somewhat more formal yet brief introduction to computational complexity theory, and it would have been informative to include a proof of why for instance the computation of the structured singular value is NP-hard. As a second issue, the authors point towards the conservatism which results from reducing or avoiding structure by over-bounding uncertainties, in order to be able to apply low-complexity algorithms. In addition to the illustrative examples, it would have been desirable to include a pointer to recent efforts for reducing this conservatism by deterministic algorithms (for example on the basis of sum-of-squares relaxations). Together with the third classical trouble of robust stability margins, namely their discontinuity in the data, the authors convincingly set the stage for the main topic of the book, the possibilities to overcome these drawbacks with the help of randomized algorithms.

The required generic problem formulation is the contents of Chapter 6, in which classical questions are related to whether a nonlinear performance function satisfies $J(\Delta)\leqslant\gamma$ for all structured uncertainty matrices $\Delta \in \mathbb{D}$ that are bounded in size as $\|\Delta\|\leqslant\rho$. The main ingredient is to reformulate this worst-case deterministic problem into the following probabilistic counterpart. It is assumed that $\boldsymbol{\Delta}$ is a matrix-valued random variable with support in $\mathbb{D}_\rho := \{\Delta \in \mathbb{D} : \|\Delta\|\leqslant\rho\}$, and the interest shifts towards guaranteeing the performance requirement $J(\boldsymbol{\Delta})\leqslant\gamma$ only with a certain (high) probability. In terms of explicitly computable and elementary examples, the authors manage to beautifully illustrate the benefits of the probabilistic counterpart for uniformly distributed $\boldsymbol{\Delta}$, if contrasted with deterministic worst-case scenarios. They somewhat too briefly address the question of how to actually choose the probability distribution of $\boldsymbol{\Delta}$, including a hint on the structure of worst-case distributions under rather strong hypotheses on their properties and the related value set.

The central idea underlying all subsequent developments is the subject of Chapter 7: Determining the probability for $J(\boldsymbol{\Delta})\leqslant\gamma$ amounts to computing a multivariable integral, which can be approximated by drawing samples of $\boldsymbol{\Delta}$ and by determining the corresponding empirical probability. The role of the weak and strong laws of large numbers is clearly pointed out to prove convergence, and the general relation to integration and maximization of functions by Monte-Carlo techniques is exhibited. It is certainly appreciated by a control readership that the authors include a short exposition of quasi Monte-Carlo methods, with its main question of finding well-distributed function evaluation grid points for optimal approximation power. These techniques are contrasted with each other in Chapter 19 when analyzing the stability and robustness of a high-speed network, where it remains a bit unfortunate that the example does not allow to draw a clear conclusion about the benefits and drawbacks of the respective sampling-schemes.

Chapter 8 surveys key conceptual algorithms for robustness analysis and synthesis, which are further elaborated on in detail in Chapters 9–12. It is initialized with analysis, including probabilistic performance verification or worst-case performance computation by sampling the uncertainties, and computing either the empirical probability or the maximum of the performance function over the drawn samples. These easy-to-implement algorithms are complemented by a priori bounds on the sample-size for the desired probabilistic guarantees, the so-called sample-complexity. The theoretical background for this distinguishing feature, if compared to classical Monte-Carlo techniques, is very accessibly developed in Chapter 9. It is without doubt a strong benefit of the book to see how elementary arguments accompanied by illustrative control examples allow to derive Bernoulli- and Chernoff-bounds from the classical Markov inequality, and how these bounds guarantee polynomiality of the suggested randomized algorithms as made precise by the authors at the end of Chapter 8.

If the performance cost $J(\Delta,\theta)$ depends as well on some design variable θ, which parameterize the to-be-constructed controller, the analysis algorithms are extended to optimized synthesis for average performance $E[J(\boldsymbol{\Delta},\theta)]$, by just minimizing the corresponding empirical mean over θ. Due to the dependence on the optimization variable θ, the estimation of the sample-complexity requires to extend classical probabilistic inequalities for individual random variables to either finite or uncountable families, and to understand under which conditions such inequalities are valid uniformly in θ. This necessitates to enter the field of statistical learning theory, for which the authors provide a nice survey in the first two sections of Chapter 10, which includes a brief exposition of the role and relevance of Vapnik–Chernovenkis- and Pollard-dimension (for binary- or continuously-valued function families), and some insights about how to work with, and estimate these quantities. The authors argue that the conservatism of the sample complexity estimates resulting from advanced theory can be circumvented by adopting as well a sampling strategy for the decision variable θ. This leads to considerably improved theoretical estimates for uncertainty sample sizes by elementary techniques as illustrated by nice examples, while the novice reader issomewhat left alone with understanding the real power of statistical learning theory for control.

A considerable part of the book is devoted to robust performance synthesis if the dependence of $J(\Delta,\theta)$ is convex in θ. On the one hand, this restricts the class of control problems to those that can indeed be exactly or approximately convexified in the controller parameters θ, such as guaranteed cost control (Chapter 11) or the synthesis of linear parameter-varying controllers (Chapter 12). On the other hand, it captures the generic robust LMI problem (Chapter 11) which requires to minimize a linear cost function of θ

over the robust LMI constraint

$$F(\Delta, \theta) \leqslant 0 \quad \text{for all } \Delta \in \mathbb{D}_\rho,$$

where $F(\Delta, \theta)$ is symmetric-valued and affine in θ. All these problems are handled by constructing some performance violation function $v(\Delta, \theta) \geqslant 0$, whose value vanishes precisely in case the desired performance specification is satisfied, and whose subgradient with respect to θ can be easily computed. One can then summarize the key idea of the suggested sequential algorithms as follows: Given $\theta^{(k)}$, sample the uncertainties until finding some $\Delta^{(i)}$ which violates performance, and update $\theta^{(k)}$ either along the direction of a subgradient of $v(\Delta^{(i)}, \theta^{(k)})$, or by relying on the standard rule as in the classical ellipsoid algorithm. The reviewer highly appreciates the careful discussion of the behavior of these algorithms under various hypothesis, and of the probabilistic guarantees even if allowing for only finitely many uncertainty samples. However, the exposition had benefited from subsuming the three concrete control problems to just one generic formulation and its algorithmic solution, which might have avoided some redundancy. On the technical side, the suggested sequential algorithms are known to be rather inefficient in practice, for example if solving nominal LMI problems. Therefore, the experienced readers might miss a discussion of the potentials to merge considerably more efficient (such as interior point) techniques from convex optimization with randomization.

Sequential techniques are only rather briefly contrasted with scenario designs in Chapter 13, in which one draws N sample uncertainties $\Delta^{(i)}$, $i = 1, \ldots, N$, and uses them all to define the constraints in a deterministic convex optimization problem. Such an algorithm is nicely demonstrated for a SISO control problem that can be reduced to robust linear programming. However, if solving robust LMI problems, the authors' claim of computational efficiency has to be taken with a grain of salt, since realistically sized control problems require N to be taken in the ten thousands for reasonable probabilistic guarantees, which is not quite tractable for presently available LMI solvers.

All algorithms rely on sampling of uncertainties, which are stochastic variables taking matrices as their values. Even if a good random number generator for $[0, 1]$ is available (such as in most computational packages), it is a non-trivial task to design one for unit balls of real or complex structured matrices. It is a unique feature of the book that the underlying theory for the construction of reliable sample generation algorithms is discussed in depth and rather accessibly in Chapters 14–18. The particular technicalities are complemented by graphical illustrations of potential pitfalls, and by concise descriptions of algorithms for easy implementation. The latter aspect should be of considerable help to further disseminate probabilistic techniques within the control community, in particular for practitioners.

The book concludes with three case studies about the principal application of randomized algorithms to high-speed networks, flexible structures, and quantized sampled-data systems. The description of the models themselves take up so much space that, unfortunately, only very little discussion is devoted to the strength and drawbacks of probabilistic versus conventional optimization techniques. Only on the basis of the stability analysis of a flexible structure, the reader can indeed judge the relation to structured singular value analysis. The reviewer would have liked to see a demonstration of the real power of randomization for a relatively large sized problem on which classical algorithms do suffer from their inherent complexity, while randomized algorithms offer a tractable polynomial-time resort.

In summary, this book provides an excellent survey of how randomization techniques are successfully applicable for advanced robust control. Whenever required, the authors provide a comprehensive, albeit sometimes a bit enumerative, survey of the most essential facts in control, probability and statistics. Much care is devoted to a rather elementary yet precise and insightful exposition of the book's main theme, the estimation of the sample-complexity for randomized analogues of computationally hard deterministic control problems. The style of writing renders it very well accessible for final year Master's students with a solid background in robust control, and for the whole control community it constitutes an excellent up-to-date foundation for entering research on probabilistic techniques.

References

Barmish, B. R. (1994). *New tools for robustness of linear systems*. New York: Macmillan.

Zhou, K., Doyle, J., & Glover, K. (1996). *Robust and optimal control*. Upper Saddle River, New Jersey: Prentice Hall.

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